Journal of Applied Mechanics and Technical Physics, Vol. 48, No. 5, pp. 686–693, 2007

EFFECT OF THE CONTACT-LINE DYNAMICS ON THE NATURAL OSCILLATIONS OF A CYLINDRICAL DROPLET

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UDC 532.5.032

The natural oscillations of a cylindrical droplet of an inviscid liquid surrounded by a different liquid and bounded in the axial direction by solid planes are studied. The motion of the contact line is described using an effective boundary condition. The dependence of the frequency and damping ratio on the capillary parameter is found. It is shown that the fundamental frequency of the translation mode vanishes beginning from a certain value of the capillary parameter. Depending on the ratio of the radial and axial dimensions of the droplet, the fundamental frequency of the axisymmetric mode and modes higher than the translation mode can vanish in a certain range of the capillary parameter. This dependence of the natural oscillation frequencies on the problem parameters allows one to determine the capillary parameter.

Key words: *cylindrical droplet, contact line, natural oscillations.*

Introduction. The mechanical equilibrium of a liquid column (liquid zone) and a jet with respect to small free capillary oscillations was studied theoretically and experimentally by Plateau [1, 2] and Rayleigh [3, 4]. In the papers cited, the limiting ratio of the height of the liquid column h and its radius R was found to be $h = 2\pi R$. For large values of h, the column becomes unstable and collapses (Rayleigh instability). Instability of a cylindrical jet surrounded by a different liquid was studied in [4]. The natural frequencies of a free liquid column are given in [5].

Interest in such configurations is motivated by their applications in various technological processes. For example, a liquid zone is used in growing semi-conductor crystals. The main emphasis has been placed on the flow in the liquid zone in the presence of heating, vertical oscillations, and a magnetic field. We note that most studies have been concerned with a cylindrical liquid column (liquid zone) surrounded by a gas, whose effect is ignored. Thus, the side surface of the column is considered free.

Sanz [6] studied the axisymmetric natural oscillations of a cylindrical liquid bridge with a fixed contact line in a vessel of finite dimensions under no gravity conditions. A comparison showed that the frequency values obtained were in good agreement with the experimental data also given in [6]. Similar studies have been performed for nonlinear natural oscillation modes [7].

Problems with the contact-line dynamics taken into account have been examined in various formulations. Most efforts have been directed toward investigating high-frequency oscillations of small amplitude (free or forced). In this case, as in many other problems of liquid oscillations, viscous and nonlinear terms in the Navier–Stokes equations can be ignored. Then, the conditions imposed on the motion of the contact line of three media is of the greatest interest.

The most widely used condition (by virtue of its simplicity) is the one employed in a study [8] of the damping of standing waves between two vertical walls. This condition assumes a linear relationship between the velocity of motion of the contact line and the contact angle (in the case of right equilibrium contact angle):

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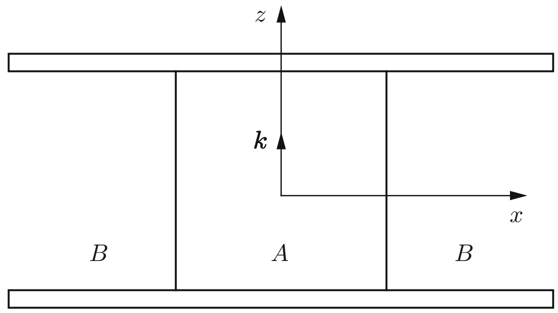

Fig. 1. Geometry of the problem: A is the droplet; B is the surrounding liquid.

$$\frac{\partial \zeta*}{\partial t} = \Lambda \boldsymbol{k} \cdot \nabla \zeta^*. \tag{1}$$

Here ζ^* is the deviation of the surface from the equilibrium position, Λ is the phenomenological constant (the so-called capillary parameter or wetting parameter), and $\boldsymbol{k}$ is the normal vector to the solid surface. We note that conditions of fixed contact line and constant contact angle are particular cases of the boundary conditions $\Lambda = 0$ and $\Lambda = \infty$, respectively. It has been shown [8] that, by virtue of the boundary condition (1), the liquid oscillations attenuate, except in the cases $\Lambda = 0$ and $\Lambda = \infty$. The same paper gives results of a qualitative comparison with experimental data.

Formula (1) well describes the results of experiments [9] with a small change in the contact angle. In addition, a qualitative comparison [8] of theoretical results with experimental data [9] has shown that surface polishing leads to a severalfold increase in the wetting parameter, i.e., the parameter Λ also characterizes the degree of surface finish of the support. Formula (1) probably results from linearization of a certain more complex boundary condition (see, for example, [10, 11]).

In most papers dealing with the contact-line dynamics, the spread of the liquid (droplet) and interaction with the support have been studied (see, for example, [11, 12]). Borkar and Tsamopoulos performed [13] a theoretical analysis of the axisymmetric natural oscillations of a liquid zone with a free side surface in a gravity field. The motion of the contact line was described using condition (1), and dissipation in a thin dynamic boundary layer on solid surfaces was studied. Dissipation due to the motion of the contact line was shown to make a major contribution to the damping. The nonaxisymmetric natural oscillations of a viscous liquid zone with a free side surface in a gravity field with a fixed contact line were studied in [14].

Formulation of the Problem. We consider the natural oscillations of a liquid droplet of density ρ_i^* surrounded by a different liquid of density ρ_e^*. Here and below, the quantities with the subscript i correspond to the droplet, and the quantities with the subscript e to the surrounding liquid. The system is bounded by two parallel solid planes (Fig. 1). The vessel is closed at infinity. In the absence of external forces, the droplet has the shape of a cylinder of radius R. The contact angle between the side surface of the droplet and the solid planes in equilibrium is equal to $\pi/2$. The distance between the bounding surfaces is equal to h.

The characteristic amplitude of oscillations of the droplet A^* is small compared to the equilibrium radius R. We assume that, on the one hand, the fundamental oscillation frequency ω^* is large enough for the viscosity can be ignored, and, on the other hand, the oscillation frequency is small enough, so that we can use the incompressibility conditions $\delta = \sqrt{\nu/\omega^*} \ll R$ and $\omega^* R \ll c$ (δ is the boundary-layer thickness, c is the sound velocity, and ν is the kinematic viscosity). For example, for a water droplet of radius 10^{-2} m, the values of the frequency ω^* meeting the indicated requirements are in a wide range from 0.1 to 100 Hz. For a free water column surrounded by a liquid with close parameters, the lowest natural oscillation frequency is $\omega^* \approx 10$ Hz.

It is convenient to use cylindrical coordinates (r^*, α, z^*), in which the droplet surface is described by the relation $r^* = R + \zeta^*(\alpha, z^*, t^*)$, where $\zeta^*(\alpha, z^*, t^*)$ is the deviation of the surface from the equilibrium position and the angle α is reckoned from the x axis.

Ignoring viscous damping, we write the Bernoulli and continuity equations in dimensionless form

$$p = -\rho(\varphi_t + \varepsilon(\nabla\varphi)^2/2), \qquad \Delta\varphi = 0. \tag{2}$$

Here the liquid velocity potential is given by the relation $\boldsymbol{v} = \nabla\varphi$; p is the pressure; the subscript at the unknown functions denotes differentiation with respect to the corresponding variables. At the interface between the liquids, the continuity condition for the normal velocity, the kinematic condition, and the balance condition for the normal stresses should be satisfied:

$$r = 1 + \varepsilon\zeta: \qquad [\boldsymbol{n}\cdot\nabla\varphi] = 0, \qquad F_t + \varepsilon\nabla\varphi\cdot\nabla F = 0, \qquad [p] = -\operatorname{div}\boldsymbol{n}. \tag{3}$$

Here the square brackets denote the jump in the quantity at the interface between the external liquid and the droplet; ζ is the deviation of the surface from the equilibrium position; $F = r - 1 - \varepsilon\zeta$; and $\boldsymbol{n} = \nabla F/|\nabla F|$ is the normal vector to the side surface of the droplet.

The velocity of motion of the contact line is proportional to the deviation of the contact angle from the equilibrium value [8]:

$$r = 1 + \varepsilon\zeta,\ z = \pm 1/2: \qquad \zeta_t = \lambda\boldsymbol{k}\cdot\nabla\zeta \tag{4}$$

(λ is the capillary parameter).

At the solid surfaces, it is necessary to impose the nonpenetration condition

$$z = \pm 1/2: \qquad \boldsymbol{k}\cdot\nabla\varphi = 0. \tag{5}$$

As the scaling quantities we use the following parameters:

$$t_0 = \sqrt{(\rho_e^* - \rho_i^*)R^3/\sigma}, \qquad v_0 = A^*\sqrt{\sigma/((\rho_e^* + \rho_i^*)R^3)}, \qquad p_0 = A^*\alpha/R^2$$

(σ is the surface tension coefficient). The boundary-value problem (2)–(5) contains five dimensionless parameters: the small relative characteristic amplitude $\varepsilon = A^*/R$, the capillary parameter $\lambda = \Lambda/\sqrt{(\rho_e^* + \rho_i^*)R^3h^2/\sigma}$, the geometrical parameter $b = R/h$, the density of the external liquid $\rho_e = \rho_e^*/(\rho_e^* + \rho_i^*)$, and the density of the liquid in the droplet $\rho_i = \rho_i^*/(\rho_e^* + \rho_i^*)$. The last two parameters are linked by the relation $\rho_i + \rho_e = 1$.

Even Modes. By the evenness of the natural oscillation modes is meant the evenness of the functions under a change of sign of the vertical coordinate z. We linearize the boundary-value problem (2)–(5) in the small parameter ε:

$$\Delta\varphi = 0, \qquad p = -\rho\varphi_t; \tag{6}$$

$$r \to \infty: \qquad \varphi \to 0; \tag{7}$$

$$r = 1: \qquad [\varphi_r] = 0, \quad \zeta_t = \varphi_r, \quad [p] = \zeta + \zeta_{\alpha\alpha} + b^2\zeta_{zz}; \tag{8}$$

$$z = \pm 1/2: \qquad \varphi_z = 0; \tag{9}$$

$$r = 1,\ z = \pm 1/2: \qquad \zeta_t = \mp\lambda\zeta_z. \tag{10}$$

In view of axial symmetry, the solution of the Laplace equation (6) is written as

$$\begin{aligned}\varphi_i &= \sum_{k=0}^{\infty}\sum_{m=1}^{\infty} a_{mk}R_{mk}^i(r)\cos(2\pi kz)\cos(m\alpha)\exp(i\Omega t),\\ \varphi_e &= \sum_{k=0}^{\infty}\sum_{m=1}^{\infty} b_{mk}R_{mk}^e(r)\cos(2\pi kz)\cos(m\alpha)\exp(i\Omega t),\end{aligned} \tag{11}$$

where $k \geqslant 1$, $R_{mk}^e(r) = K_m(2\pi bkr)$ at $k \geqslant 1$, I_m and K_m are modified Bessel functions, and Ω is the natural oscillation frequency. Using the kinematic condition and the balance condition for the normal stress (8), we seek the function of the surface deviation $\zeta(\alpha, z, t)$ in the form

$$\begin{aligned}\zeta = \Big(\sum_{m=0}^{\infty}\sum_{k=0}^{\infty} c_{mk}\cos(2\pi kz)\cos(m\alpha) + d_1 z^2\cos(\alpha) + d_2\cos\Big(\frac{z}{b}\Big)\\ + \sum_{m=2}^{\infty} e_m\cosh\Big(\frac{\sqrt{m^2-1}}{b}\,z\Big)\cos(m\alpha)\Big)\exp(i\Omega t),\end{aligned} \tag{12}$$

where the last three terms are partial solutions which follow from the third boundary condition in (8). We note that in [13] these terms were ignored. Substituting solutions (11) and (12) into (6)–(10), we obtain a spectral-amplitude problem, whose eigenvalues are values of the natural oscillation frequency Ω. From the solution of this problem, it follows that the eigenvalues are found from the equations

$$i\sum_{k=1}^{\infty}\frac{4b\Omega^3}{\gamma_{0k}^2(\omega_{0k}^2-\Omega^2)}+i2b\Omega-i\Omega\cot\left(\frac{1}{2b}\right)+\lambda\,\frac{1}{b}=0,\qquad m=0,$$

$$\sum_{k=1}^{\infty}\frac{\Omega^4}{\pi^2k^2(\omega_{1k}^2-\Omega^2)}+\frac{1}{6}\,\Omega^2-i\lambda\Omega-2b^2=0,\qquad m=1,\tag{13}$$

$$i\Omega^3\Big(\frac{2b}{\gamma_{m0}(\omega_{m0}^2-\Omega^2)}+\sum_{k=1}^{\infty}\frac{4b\gamma_{m0}}{\gamma_{mk}^2(\omega_{mk}^2-\Omega^2)}\Big)+\lambda\,\frac{\gamma_{m0}}{b}+i\Omega\coth\left(\frac{\gamma_{m0}}{2b}\right)=0,\qquad m\geqslant 2,$$

where

$$\gamma_{mk}^2=m^2-1+4\pi^2b^2k^2,$$

$$\omega_{mk}^2=\gamma_{mk}^2\,\frac{R_{mkr}^i(1)}{F_{mk}},\qquad F_{mk}=\rho_iR_{mk}^i(1)-\rho_e\,\frac{R_{mkr}^i(1)}{R_{mkr}^e(1)}\,R_{mk}^e(1),$$

ω_{mk} are the natural oscillation frequencies of the freely moving cylindrical droplet [5].

The complex algebraic equations (13) have complex solutions, which leads to oscillation damping due to dissipation on the contact line.

Odd Modes. By analogy with the solution for the even modes, the solution of the Laplace equation (6) is written as

$$\varphi_i=\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}a_{mk}R_{mk}^i(r)\sin\left[(2k+1)\pi z\right]\cos(m\alpha)\exp(i\Omega t),$$

$$\varphi_e=\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}b_{mk}R_{mk}^e(r)\sin\left[(2k+1)\pi z\right]\cos(m\alpha)\exp(i\Omega t),$$

where $R_{mk}^i(r)=I_m((2k+1)\pi br)$, $R_{mk}^e(r)=K_m((2k+1)\pi br)$, and I_m and K_m are modified Bessel functions.

Using the results (12) obtained above, we write the function of the surface deviation as

$$\zeta=\Big[\sum_{m=0}^{\infty}\sum_{k=0}^{\infty}c_{mk}\sin\left[(2k+1)\pi z\right]\cos(m\alpha)+d_1z\cos(\alpha)+d_2\sin\left(\frac{z}{b}\right)$$

$$+\sum_{m=2}^{\infty}e_m\sinh\left(\frac{\sqrt{m^2-1}}{b}\,z\right)\cos(m\alpha)\Big]\exp(i\Omega t).$$

Calculations similar to those for the even modes yield the following equations for the natural oscillation frequencies of odd modes:

$$\sum_{k=0}^{\infty}\frac{i4b\Omega^3}{\delta_{0m}^2(\omega_{0k}^2-\Omega^2)}+i\Omega\tan\left(\frac{1}{2b}\right)+\lambda\,\frac{1}{b}=0,\qquad m=0,$$

$$\sum_{k=0}^{\infty}\frac{4\Omega^2}{(2k+1)^2\pi^2(\omega_{1k}^2-\Omega^2)}+\frac{1}{2}+\frac{\lambda}{i\Omega}=0,\qquad m=1,\tag{14}$$

$$\sum_{k=0}^{\infty}\frac{4\Omega^2b\delta_{m0}}{\delta_{mk}^2(\omega_{mk}^2-\Omega^2)}+\frac{\lambda\delta_{m0}}{i\Omega b}+\tanh\left(\frac{\delta_{m0}}{2b}\right)=0,\qquad m\geqslant 2.$$

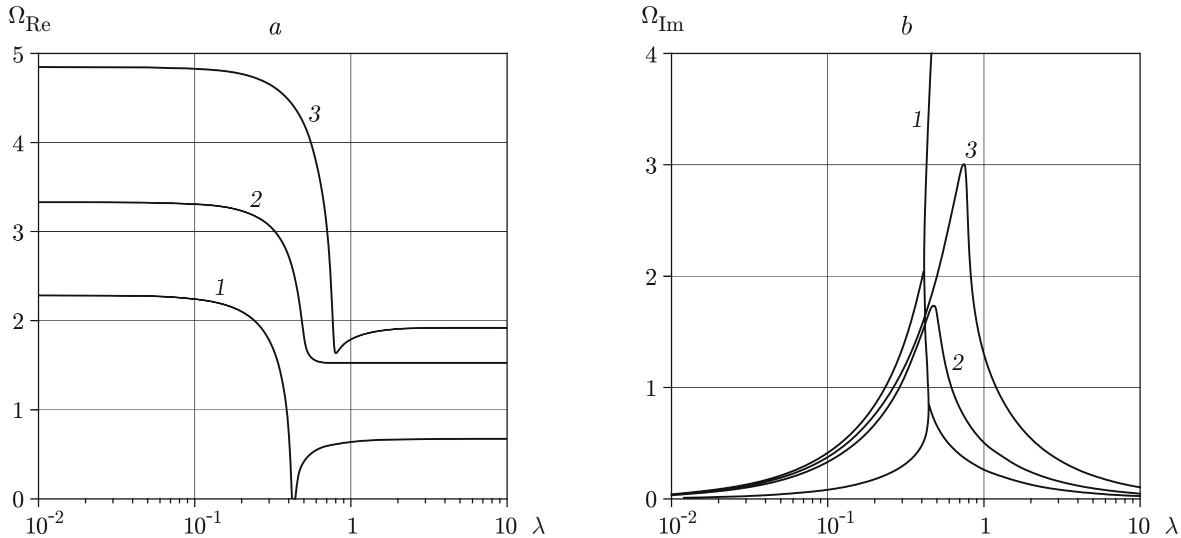

Fig. 2. Natural oscillations frequency (a) and damping ratio (b) versus capillary parameter for the frequencies Ω_{11} and Ω_{01}: curves 1 and 3 refer to the frequency Ω_{01} for $b = 0.4$ (1) and $b = 0.6$ (3); curve 2 refer to the frequency Ω_{11} for $\rho_i = 0.75$ and $b = 0.4$.

Here

$$\delta^2_{mk} = m^2 - 1 + (2k+1)^2\pi^2 b^2,$$

$$\omega^2_{mk} = \delta^2_{mk}\,\frac{R^i_{mkr}(1)}{F_{mk}}, \qquad F_{mk} = \rho_i R^i_{mk}(1) - \rho_e\,\frac{R^i_{mkr}(1)}{R^e_{mkr}(1)}\,R^e_{mk}(1).$$

Results. Equations (13) and (14) were solved numerically using the two-dimensional secant method. Figure 2 shows the real part $\Omega_{\rm Re}$ (oscillation frequency) and imaginary part $\Omega_{\rm Im}$ (damping ratio) of the complex natural frequency Ω for the oscillation modes Ω_{01} (i.e., for $m = 0$ and $k = 1$, where m is the radial mode number and k is the wavenumber) and Ω_{11}. Here and below, in the frequency indices, consecutive numbering of the wavenumber is used: even values of k correspond to even modes [the solution of Eqs. (13)] and odd values of k correspond to odd modes [the solution of Eqs. (14)]. In a certain range of λ, the real part of the frequency Ω_{01} can vanish, depending on the value of the geometrical parameter b (Fig. 2a). For large values of b, this range is absent. As b decreases, the value of λ increases and tends to infinity for $b = 1/\pi$. The vanishing of $\Omega_{\rm Re}$ corresponds to the bifurcation of the branch of the increment $\Omega_{\rm Im}$ (curve 1 in Fig. 2b). For large values of the capillary parameter λ, the real part of the frequency Ω_{01} vanishes, according to the data in Fig. 2a and according to the expression for ω_{01} for $b \leqslant 1/\pi$. This is due to Rayleigh instability of the liquid column for $h = 2\pi R$, i.e., for $b = 1/(2\pi)$. Thus, for $b = 1/\pi$, the thickness of the layer is equal to the Rayleigh instability half-wavelength.

Figure 3 shows a curve of the damping increment $\Omega_{\rm Im}$ versus the geometrical parameter b for $m = 0$. The increment takes negative values for $b < 1/\pi$, which corresponds to the occurrence of monotonic instability because $\Omega_{\rm Re} = 0$.

From Fig. 2 it follows that as λ increases, the frequency decreases monotonically, the damping ratio has a maximum for a finite value of the capillary parameter and tends to zero as $\lambda \to 0$ and $\lambda \to \infty$. In the first case ($\lambda \to 0$), the following expressions are valid for the even modes:

$$-8b\sum_{k=1}^{\infty}\frac{\Omega^2_{0\,\rm Im}\Omega_{0\,\rm Re}}{\gamma^2_{0k}(\omega^2_{0k} - \Omega^2_{\rm Re})} + \Omega_{0\,\rm Im}\cot\left(\frac{1}{2b}\right) + \lambda\frac{1}{b} = 0,$$

$$4\Omega^2_{1\,\rm Re}\Omega_{1\,\rm Im}\sum_{k=1}^{\infty}\frac{\omega^2_{1k}}{\pi^2 k^2(\omega^2_{1k} - \Omega^2_{1\,\rm Re})^2} + \frac{1}{3}\,\Omega_{1\,\rm Im} - \lambda = 0,$$

$$3\Omega^2_{m\,\rm Re}\Omega_{m\,\rm Im}\sum_{k=1}^{\infty}\frac{2b^2\omega^2_{mk}\gamma_{m0}}{\gamma^2_{mk}(\omega^2_{mk} - \Omega^2_{m\,\rm Re})^2} + \Omega_{m\,\rm Im}\coth\left(\frac{\gamma_{m0}}{2b}\right) = \lambda\,\frac{\gamma_{m0}}{b}.$$

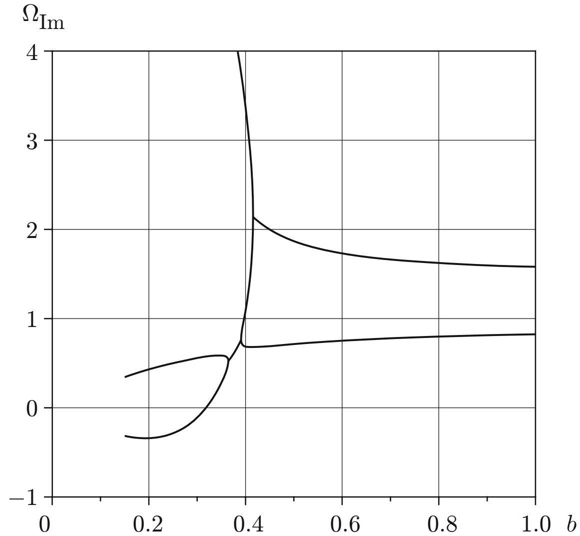

Fig. 3. Damping ratio versus geometrical parameter for the frequency Ω_{01} for $\lambda = 0.44$ and $\rho_i = 0.75$.

Here $\Omega_{0\,\rm Re}$, $\Omega_{1\,\rm Re}$, and $\Omega_{m\,\rm Re}$ are determined from the relations

$$\sum_{k=1}^{\infty} \frac{2b\Omega_{0\,\rm Re}^2}{\gamma_{0k}^2(\omega_{0k}^2 - \Omega_{0\,\rm Re}^2)} - \cot\left(\frac{1}{2b}\right) = 0, \qquad \sum_{k=1}^{\infty} \frac{\Omega_{1\,\rm Re}^4}{\pi^2 k^2(\omega_{1k}^2 - \Omega_{1\,\rm Re}^2)} + \frac{1}{6}\,\Omega_{1\,\rm Re}^2 = 2b^2,$$

$$\Omega_{m\,\rm Re}^2 \sum_{k=1}^{\infty} \frac{2b^2\omega_{mk}^2\gamma_{m0}}{\gamma_{mk}^2(\omega_{mk}^2 - \Omega_{m\,\rm Re}^2)^2} + \coth\left(\frac{\gamma_{m0}}{2b}\right) = 0.$$

In the second case ($\lambda \to \infty$), the damping ratios are expressed as

$$\Omega^{(1k)} = \frac{\omega_{1k}^2}{2\lambda\pi^2 k^2}, \qquad \Omega^{(mn)} = \frac{\omega_{mk}^2}{\lambda\gamma_{mk}^2}, \quad m \neq 1.$$

In this case, the natural oscillation frequencies of the droplet coincide with the natural frequencies of the freely moving cylindrical droplet [5].

As for the even modes, with increasing λ, the odd mode frequency decreases monotonically, the damping ratio has a maximum for a finite value of the capillary parameter and tends to zero as $\lambda \to 0$ and $\lambda \to \infty$. In the first case, the following expressions are valid:

$$4b\sum_{k=0}^{\infty} \frac{3\Omega_{0\,\rm Im}^2\Omega_{0\,\rm Re}}{\delta_{0k}^2(\omega_{0k}^2 - \Omega_{\rm Re}^2)} + \Omega_{0\,\rm Re}\tan\left(\frac{1}{2b}\right) = \lambda\,\frac{1}{b}, \qquad \sum_{k=0}^{\infty} \frac{4\Omega_{1\,\rm Re}^2\Omega_{1\,\rm Im}}{\pi^2(2k+1)^2(\omega_{1k}^2 - \Omega_{\rm Re}^2)} + \frac{1}{2}\,\Omega_{1\rm Re} = \lambda,$$

$$3\Omega_{m\,\rm Re}^2\Omega_{m\,\rm Im}\sum_{k=0}^{\infty} \frac{4b\gamma_{m0}}{\delta_{mk}^2(\omega_{mk}^2 - \Omega_r^2)} + \Omega_{m\,\rm Im}\tanh\left(\frac{\gamma_{m0}}{2b}\right) = \lambda\,\frac{\gamma_{m0}}{b}.$$

Here $\Omega_{0\,\rm Re}$, $\Omega_{1\,\rm Re}$, and $\Omega_{m\,\rm Re}$ are determined from the relations

$$\sum_{k=0}^{\infty} \frac{4b\Omega_{0\,\rm Re}^2}{\delta_{0k}^2(\omega_{0k}^2 - \Omega_{0\,\rm Re}^2)} + \tan\left(\frac{1}{2b}\right) = 0, \qquad \sum_{k=0}^{\infty} \frac{\Omega_{1\,\rm Re}^2}{(2k+1)^2\pi^2(\omega_{1k}^2 - \Omega_{1\,\rm Re}^2)} + \frac{1}{2} = 0,$$

$$\sum_{k=0}^{\infty} \frac{4b\Omega_{m\,\rm Re}^2\gamma_{m0}}{\delta_{mk}^2(\omega_{mk}^2 - \Omega_{m\,\rm Re}^2)} + \tanh\left(\frac{\gamma_{m0}}{2b}\right) = 0.$$

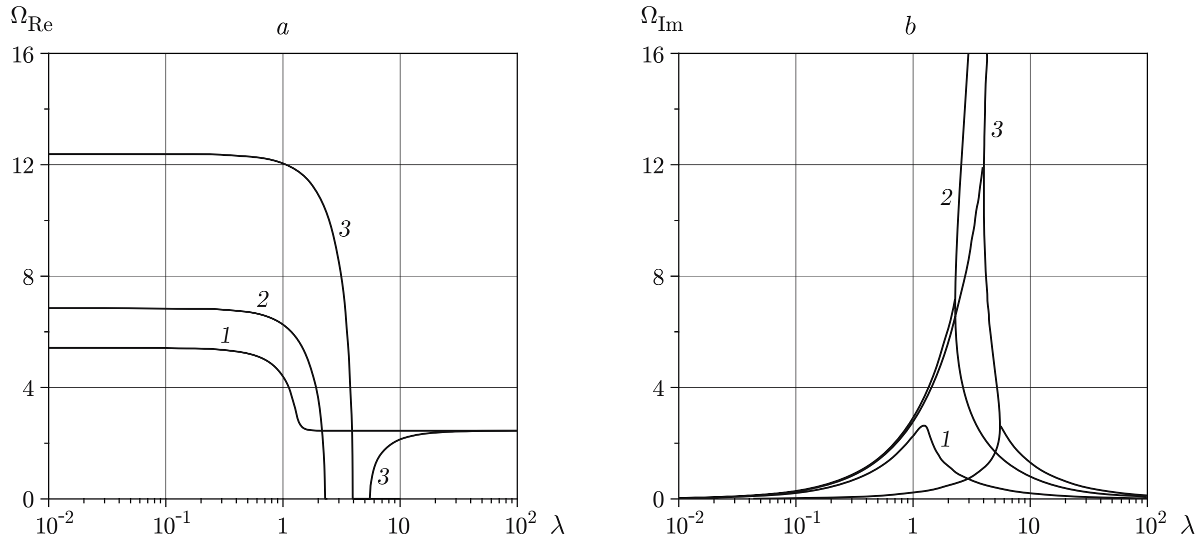

Fig. 4. Real part (a) and imaginary part (b) of the frequency versus capillary parameter: curves 1 and 3 refer to the frequency Ω_{20} for $b = 1$ (1) and $b = 2.5$ (3) and curve 2 refers to the frequency Ω_{10} for $b = 2$.

In the second case ($\lambda \to \infty$), the damping ratios are expressed as

$$\Omega^{(1k)} = \frac{2\omega_{1k}^2}{\lambda(2k+1)^2\pi^2}, \qquad \Omega^{(mk)} = \frac{\omega_{mk}^2}{\lambda\delta_{mk}^2}, \quad m \neq 1.$$

The first translation mode describes the displacement of the droplet as a whole. In the case considered (with the contact-line dynamics taken into account) the displacement is larger in the central part of the column than near the ends. Elastic forces cause the droplet to take the original shape, resulting in the return motion of its center of mass. As the capillary parameter increases, the shift between the center and periphery of the droplet surface decreases. For a certain value of the capillary parameter, the difference in the value of the shift disappears and the first eigenmode frequency vanishes (Fig. 4a). For large values of λ, the damping ratio takes two values (Fig. 4b). For the frequencies Ω_{m0} ($m = 2, 3, \ldots$) there exists a certain range of the capillary parameter λ in which the real parts of these frequencies vanish. The length of this range increases with increasing parameter b (Fig. 4), in contrast to the axisymmetric mode. From Fig. 4 it follows that the natural oscillation frequency of the droplet increases as the parameter b increases (i.e., as the equilibrium radius increases or as the droplet height decreases).

Conclusions. The oscillations of a cylindrical liquid droplet surrounded by a different liquid and enclosed between two solid surfaces were studied. The contact-line dynamics was taken into account: the velocity of motion of the contact line was assumed to be proportional to the deviation of the contact angle from the equilibrium value. The proportionality coefficient, the so-called capillary parameter (wetting parameter), characterizes the properties of the liquid and the support material. The equilibrium contact angle is equal to $\pi/2$.

It was shown that an increase in the capillary parameter leads to a reduction in the natural oscillation frequency. The droplet moving freely on the solid surfaces has the lowest natural frequency.

The axisymmetric eigenmode frequency can vanish in a certain range of the capillary parameter λ, depending on the value of the geometrical parameter b. For $b \leqslant 1/\pi$ and a certain characteristic value of λ, the frequency vanishes and the increment becomes negative, which corresponds to the occurrence of Rayleigh instability. As the value of b increases, the length of this range decreases.

As the capillary parameter increases, the frequency of the translation mode decreases, and for a certain value of λ, it vanishes for any b.

For the fundamental frequencies with the azimuthal numbers $m = 2, 3, \ldots$ there exists a certain range of the capillary parameter λ in which the real parts of these frequencies also vanish. However, in contrast to the axisymmetric mode, the length of this range increases with increasing parameter b.

Thus, one can choose the droplet radius-to-height ratio such that the characteristic frequency of any mode is equal to zero, and, ultimately, to determine the capillary parameter λ.

This work was supported by the Russian Foundation of Basic Research (Grant No. 04-01-00442), American Fund of Civil Research and Development (Grant No. PE-009-0/B2M409), and Program of the President of the Russian Federation on the State Support of Leading Scientific Schools (Grant No. NSh-1981.2003.1).

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